Elementary Math for Computer Science

with Python

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1
Variables

Image credit: Carol M. Highsmith

> The height of a tree could be represented by a variable because it changes over time.

Introduction

Some things never change: the number of inches in a foot; the number of petals on a tulip; the speed of light. Quantities like these are known as constants.

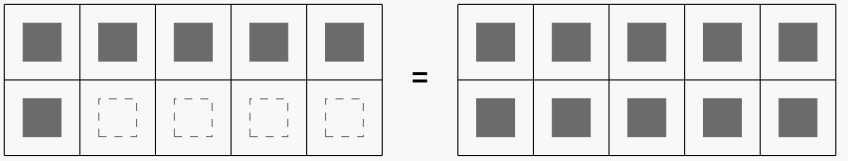
Other things change frequently: the day of the week; the height of a tree; a person's age. Quantities like these are best represented

Examples

Addition with variables

Q: Find the value of the variable:

$$6 + x = 10$$



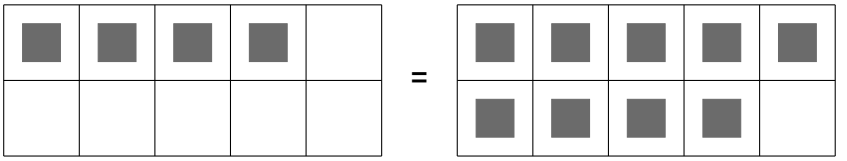
A: Using the ten-frames above, we can see that 4 tiles must be added to the left side in order to match the total of 10 tiles on the right side.

In other words, since $6 + 4 = 10$, the value of x must be 4.

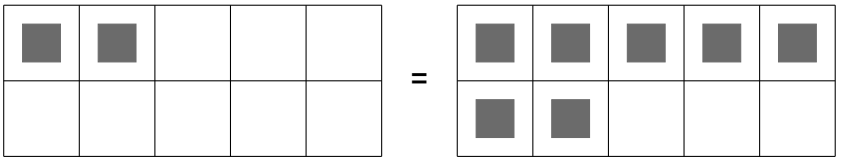
Practice

1. Use the tile models to find the value of the variable.

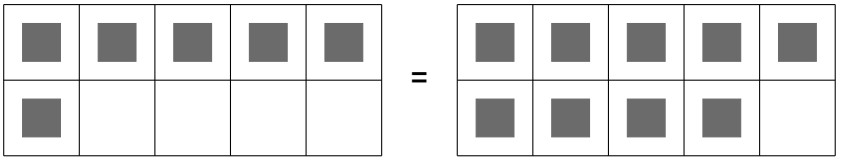
a. $4 + a = 9$



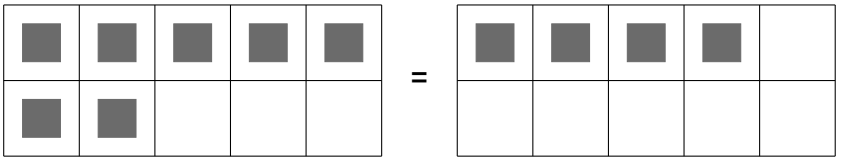
b. $b + 2 = 7$



c. $6 + c = 9$



d. $7 - d = 4$



The climate in Austin, Texas is much warmer than in Boise, Idaho.

Image credit: Carol M. Highsmith

7. **The coldest recorded temperature** in Boise, Idaho occurred on December 23rd, 1990. On that day, the temperature reached -25 °F. The coldest recorded temperature in Austin, Texas occurred on February 2nd, 2011. On that day, the temperature reached 17 °F. How many degrees warmer is the record-low in Austin than the record-low in Boise?

8. **To create branches in a computer program** that lead to different outcomes, a programmer will often use "**if-then**" constructs. For example: *If* the player completes a challenge before the time expires, *then* the player earns a perfect score. Complete the following "if-then" constructs about absolute values:

   a. If a number is positive, then its absolute value is...

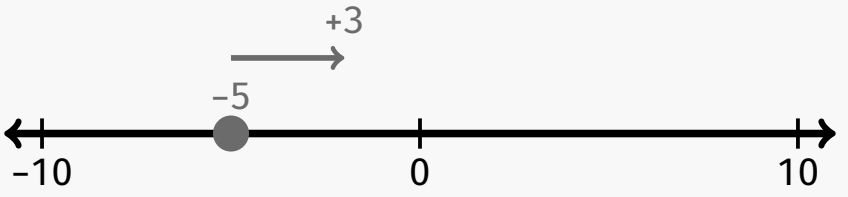
   b. If a number is negative, then its absolute value is...

   c. If two numbers have the same absolute value, then the numbers are...

Examples

Adding negative numbers

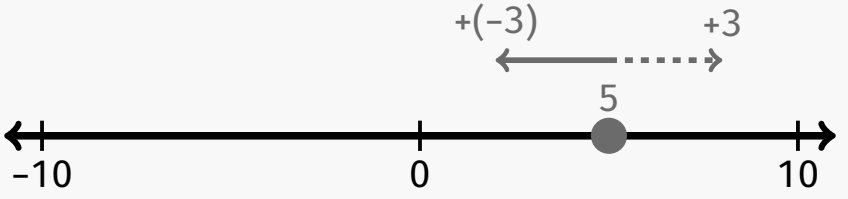
Q: Evaluate: −5 + 3



A: Adding a positive results in a move to the right on the number line. Thus:

$$-5 + 3 = -2$$

Q: Evaluate: 5 + (−3)



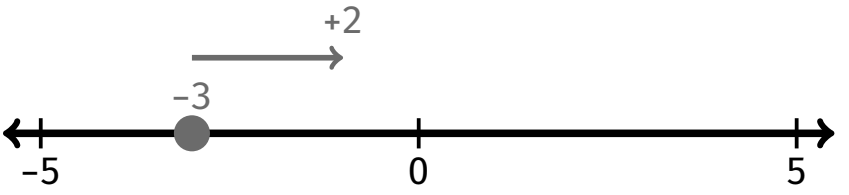
A: Adding a negative results in a move to the left on the number line. Thus:

$$5 + (-3) =$$
$$5 - 3 = 2$$

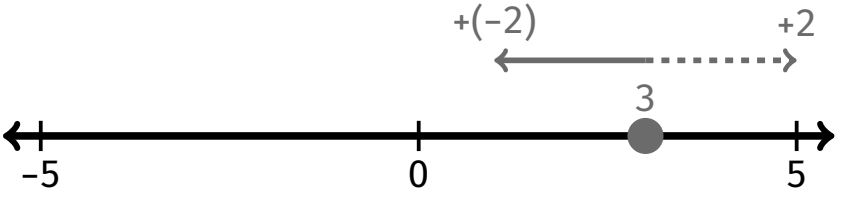
Practice

1. Use the number lines to find each sum.

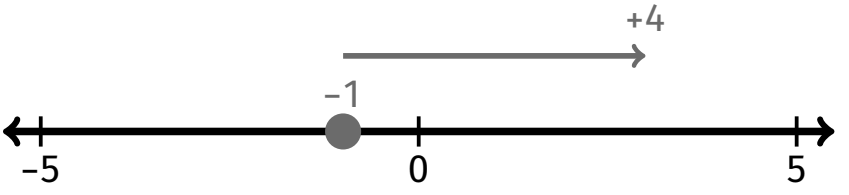
a. −3 + 2



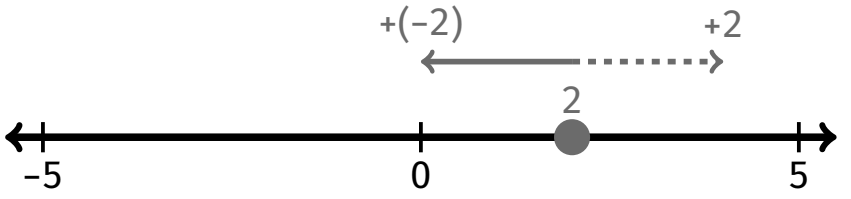
b. 3 + (−2)



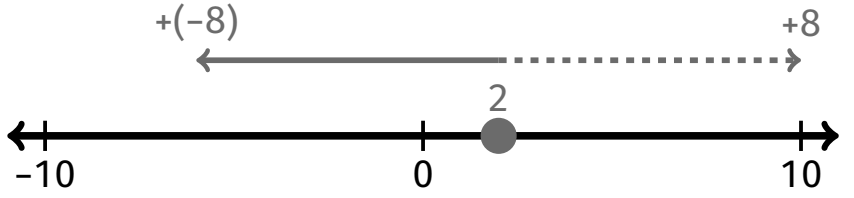
c. −1 + 4



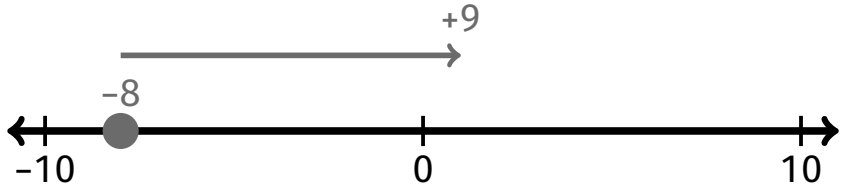
d. 2 + (−2)



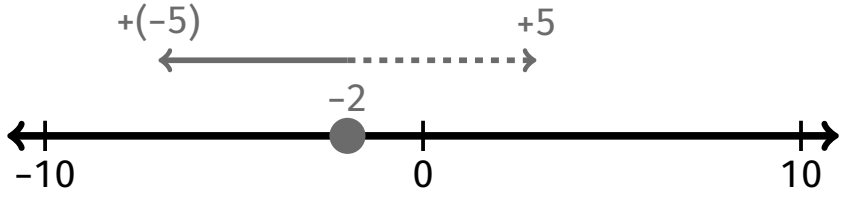
e. 2 + (−8)



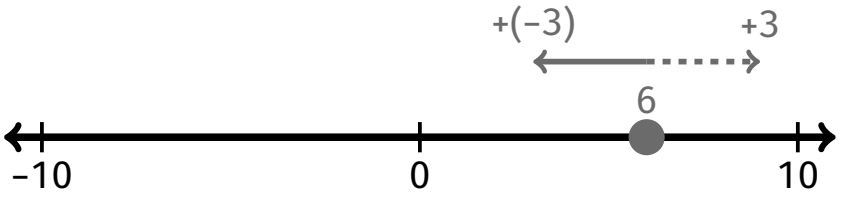
f. −8 + 9



g. −2 + (−5)



h. 6 + (−3)



2. Find the sum. If you get stuck, try plotting points on a number line.

a. −1 + 1

b. −2 + 4

c. 3 + (−1)

d. 4 + (−3)

e. −3 + (−5)

f. −7 + 6

g. 8 + (−9)

h. −4 + 12

i. 6 + (−10)

j. −11 + 5

k. 17 + (−8)

l. 13 + (−20)

m. −22 + 27

n. 15 + (−30)

o. 21 + (−32)

3. Sperm whales are known to dive incredibly deep in search of squid. Imagine that a sperm whale dives to a depth of 2,100 ft (-2,100 ft). Still searching for squid, it dives a further 1,080 ft to its maximum observed diving depth. Use integer addition to find the maximum observed diving depth of a sperm whale.

24 Game comprehension questions

1. How would you print the first item from a list named "`movies`"?
2. How would you print every item from a list named "`movies`"?
3. Describe the output from the following code:

```
answer = "3 + 4 * 5"
print(answer)
print(eval(answer))
```

Challenge!

After completing *24 Game* in Python, try these additional challenges:

1. Change the game to the *36 Game*.
2. Allow the user to keep guessing until they win the game.
3. Write a program that selects a random name from a list of five names.
4. In our *24 Game*, it is possible for the user to trick the computer by entering numbers not given or by omitting some of the given numbers. For a solution to this problem, see *Extension: 24 Game in Python* in the appendix.